

Linear Programming

Example: You are stenciling wooden boxes to sell at a fair, and want to make the most money possible. It takes you 2 hours to stencil a small box and 3 hours to stencil a large box, and you can only work for a maximum of 15 hours.

s - small box
L - large box

Write an appropriate inequality: $2S + 3L \leq 15$

You want to sell a minimum of 6 boxes total.

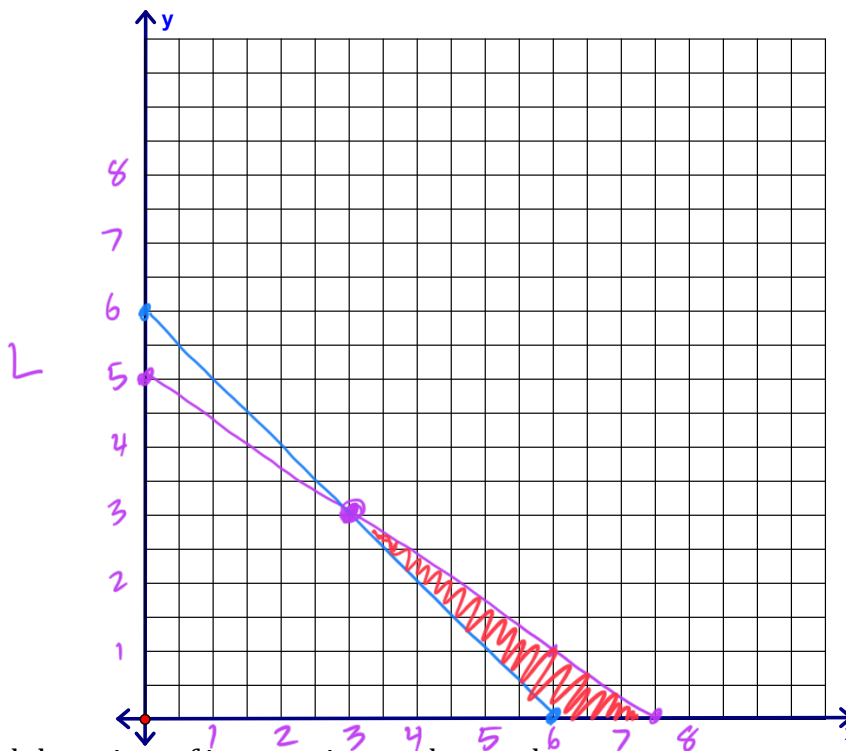
Write an appropriate inequality: $S + L \geq 6$

Your **profit** is based on \$10 for a small box and \$20 for a large box.

Write an appropriate equation: $P = 10S + 20L$

Step 1: Graph the inequalities:

since you are dealing with a real life problem, you are automatically in the positive answers (you cannot make negative boxes) so keep in mind that your first two boundaries are the x-axis and the y-axis.



$x \geq 0$
 $y \geq 0$

$L \leq -\frac{2}{3}S + 5$ ↓

$L \geq -S + 6$ ↑

Step 2: Find the points of intersection on the graph:

$(3, 3)$ $(6, 0)$ $(7.5, 0)$
s, L s, L s, L

$P = 10(3) + 20(3) = 90$ #
 $P = 10(6) + 20(0) = 60$ #
 $P = 10(7.5) + 20(0) = 75$ #

Step 3: Plug your points of intersection into the equation $P =$ _____

Step 4: Based on the values above, which point answers the question?
(Since they are looking for a maximum, which is the maximum?)

Ex 2) The junior class plans to raise money by selling two sizes of fruit baskets. The class will purchase small baskets for \$10 and large baskets for \$15. The class can afford to spend up to \$1200 to buy the baskets. The class president states that the club will not sell more than 100 baskets.

→ The class wants to **determine how many of each type of basket** to sell in order to maximize profit.

a) Assign variables. Hint: Look at what you are solving for! Be sure to clearly define the variables using “let” statements.

S = # small baskets

L = # large baskets

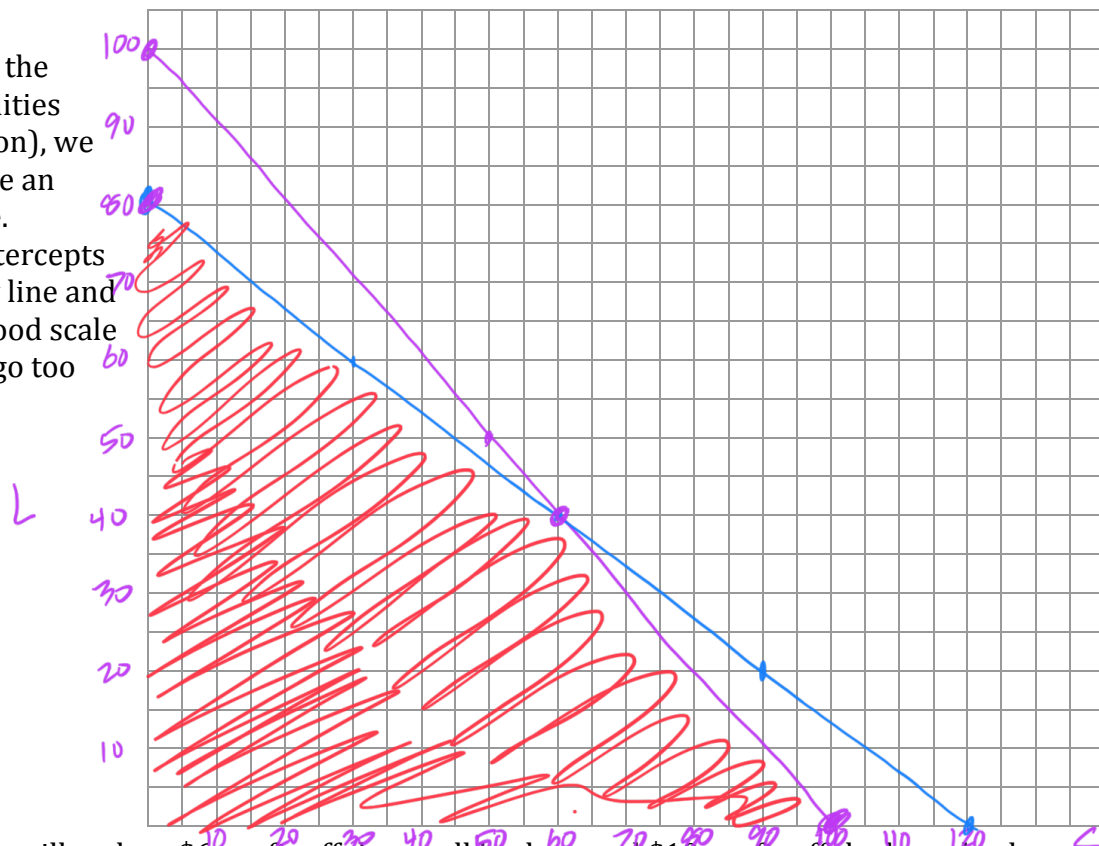
b) Write a system of inequalities to represent the constraints.

S ≥ 0 S + L ≤ 100 → L ≤ -S + 100

L ≥ 0 10S + 15L ≤ 1200 → L ≤ - $\frac{2}{3}$ S + 80

c) Graph the feasible region and label all of the corner points!

In order to graph the system of inequalities (the feasible region), we need to determine an appropriate scale. Determine the intercepts of each boundary line and think of what a good scale would be...don't go too big!



d) The class will make a \$6 profit off the small baskets and \$10 profit off the large baskets. Write the objective function for the class's total profit.

P = 6S + 10L

e) What should the club do to maximize profit?

(0, 80) 6(0) + 10(80) = \$800
(60, 40) 6(60) + 10(40) = \$760
(100, 0) 6(100) + 10(0) = \$600

Ex) A small company produces designer wall wreaths and designer candle sticks. They sell their products through a chain of specialty stores. The company is to supply the stores with a total of no more than 100 wreaths and candle sticks per day. The store guarantees that they will sell at least 10 and no more than 60 wreaths per day and at least 20 candle sticks per day.

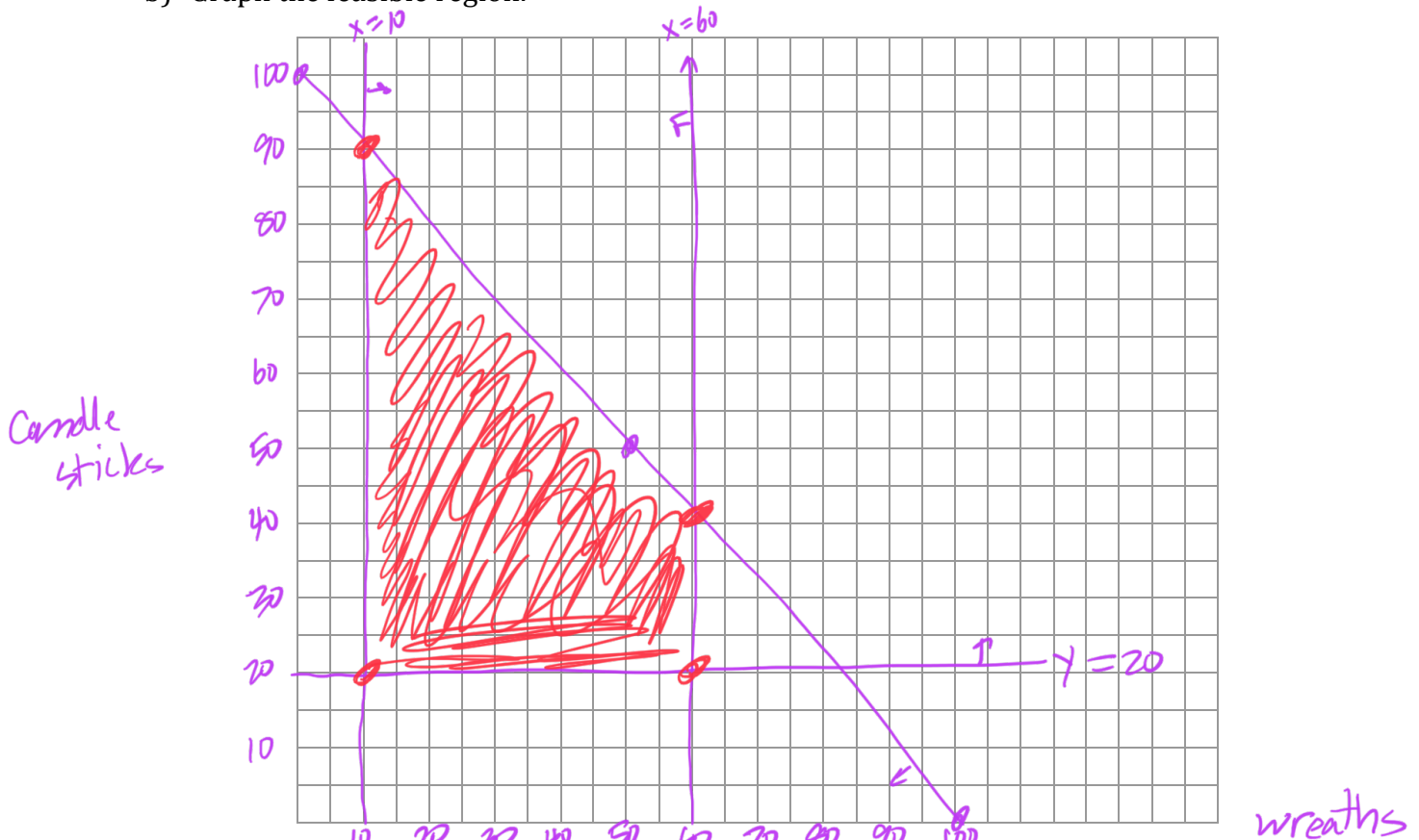
- a) Let x be the number of wreaths and let y be the number of candle sticks. Write a system of inequalities to represent the constraints.

$$x + y \leq 100 \rightarrow y \leq -x + 100$$

$$10 \leq x \leq 60$$

$$y \geq 20$$

- b) Graph the feasible region.



- c) The company makes a profit of \$10 on each wreath and a profit of \$12 on each candle sticks. Write an objective function for the company's total profit, P , from the sales of wreaths and candle sticks.

$$P = 10x + 12y$$

- d) What combination of wreaths and candle sticks sold would maximize the company's profit?

$(10, 90)$	$10(10) + 12(90) = 1180$
$(10, 20)$	$10(10) + 12(20) = 340$
$(60, 20)$	$10(60) + 12(20) = 840$
$(60, 40)$	$10(60) + 12(40) = 1080$